Transport through a single Anderson impurity coupled to one normal and two superconducting leads

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Abstract. We study the interplay between the Kondo and Andreev-Josephson effects in a quantum dot coupled to one normal and two superconducting (SC) leads. In the large gap limit, the low-energy states of this system can be described exactly by a local Fermi liquid for the interacting Bogoliubov particles. The phase shift and the renormalized parameters for the Bogoliubov particles vary depending on the Josephson phase ϕ between the two SC leads. We explore the precise features of a crossover that occurs between the Kondo singlet and local Cooper-pairing states as ϕ varies, using the numerical renormalization group approach.

1. Introduction

The Kondo effect in quantum dots (QD) connected to superconducting (SC) leads has been studied intensively in these years, experimentally [1–4] and theoretically [5–15]. The competition between the strong Coulomb repulsion and superconductivity gives rise to the quantum phase transition (QPT) between a nonmagnetic singlet and magnetic doublet ground states. It also causes the Andreev scattering at an interface between the QD and a normal lead, and affects substantially transport properties near the QPT, or the corresponding crossover region.

In the present work, we consider a single QD coupled to one normal and two SC leads as illustrated in Fig. 1, and study the interplay of the Kondo, Andreev, and Josephson effects, by using the Wilson numerical renormalization group (NRG) approach. We show that three phase variables, the phase shift δ for the interacting Bogoliubov particles, the Bogoliubov angle Θ_B at the interface between the QD and normal lead, and the Josephson phase ϕ between the two SC leads play a central role, and determine the low-temperature properties in the large SC gap limit.

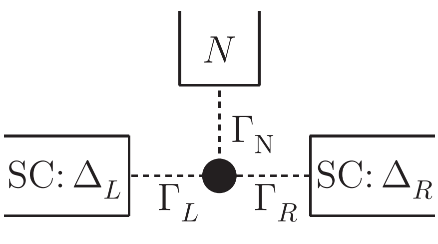

Figure 1. Anderson impurity (•) coupled to one normal (N) and two superconducting leads (L,R): $\Gamma_\nu \equiv \pi\rho v_\nu^2$ with ρ the density of states of the lead ($\nu = L, R, N$), and v_ν the tunneling matrix element. The SC gaps $\Delta_{L/R} = |\Delta_{L/R}|\, e^{i\theta_{L/R}}$ give rise to the Andreev scattering and Josephson effect.

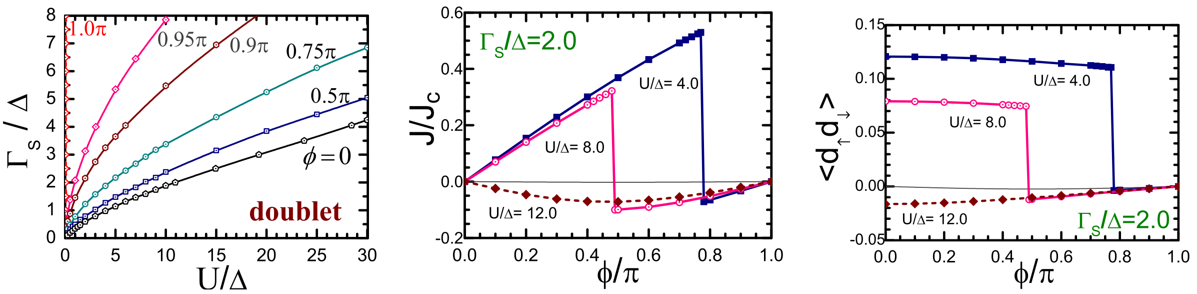

Figure 2. The left panel shows the phase boundary between the non-magnetic singlet (upper side) and magnetic doulbet (lower side) ground states in the U vs Γ_S plane for several values of $\phi \equiv \theta_R - \theta_L$. The middle and right panels show the Josephson current J and $\langle d_\uparrow d_\downarrow \rangle$ as functions of ϕ for $\Gamma_S = 2.0\Delta$ for several U. The critical current is given by $J_C = e\Delta/\hbar$ for finite Δ.

2. Model

We start with a single impurity Anderson model of the form,

$$H = \xi_d \left(n_d - 1\right) + \frac{U}{2}\left(n_d - 1\right)^2 + \sum_{\nu=N,L,R} \sum_{\sigma} v_\nu \left(\psi^\dagger_{\nu,\sigma} d_\sigma + d^\dagger_\sigma \psi_{\nu,\sigma}\right) + H_{\text{lead}} + H_{\text{BCS}} \,, \tag{1}$$

$$H_{\text{lead}} = \sum_{\nu=N,L,R} \sum_{k,\sigma} \epsilon_k \, c^\dagger_{\nu,k\sigma} c_{\nu,k\sigma}, \qquad H_{\text{BCS}} = \sum_{\alpha=L,R} \sum_{k} \left(\Delta_\alpha \, c^\dagger_{\alpha,k\uparrow} c^\dagger_{\alpha,-k\downarrow} + \text{H.c.}\right). \tag{2}$$

Here, $\xi_d \equiv \epsilon_d + U/2$, U the Coulomb interaction, $n_d \equiv \sum_\sigma d^\dagger_\sigma d_\sigma$, and $d^\dagger_\sigma$ ($c^\dagger_{\nu,k\sigma}$) the creation operator for an electron with energy ϵ_d (ϵ_k) and spin σ in the quantum dot (lead). The QD and leads are coupled via v_ν, and $\psi_{\nu,\sigma} \equiv \sum_k c_{\nu,k\sigma}/\sqrt{\mathcal{N}_\nu}$ with $\mathcal{N}_\nu$ the normalization factor one-particle states in the leads. We assume that $\Gamma_\nu \equiv \pi\rho v_\nu^2$ and the density of states ρ are independent of the frequency ω. The complex superconducting gap $\Delta_{L/R} = |\Delta_{L/R}| e^{i\theta_{L/R}}$ gives rise to the Josephson current between the two SC leads for finite $\phi \equiv \theta_R - \theta_L$. Since this Hamiltonian still has a number parameters to be explored, in this report we concentrate on the half-filled case $\epsilon_d = -U/2$, assuming a symmetric junction with $\Gamma_L = \Gamma_R$ ($\equiv \Gamma_S/2$) and $|\Delta_L| = |\Delta_R|$ ($\equiv \Delta$).

3. Two SC terminal case without the normal lead $\Gamma_N = 0$

We first of all consider a simpler $\Gamma_N = 0$ case, where the QD is coupled only to the two SC leads, in order to see the competition between the Kondo and Josephson effects [10–14]. Figure 2 shows the NRG results for the ground state properties. We see that the region of the magnetic doublet state, which emerges for large U/Δ and small Γ_S/Δ, expands in the parameter space as the Josephson phase ϕ increases. This indicates that ϕ disturbs the Kondo screening. At the QPT, the Josephson current and the SC correlation $\langle d_\uparrow d_\downarrow \rangle$ at the impurity site vary discontinuously, and take small negative values in the magnetic doublet phase for finite Δ.

4. Low-energy properties of the QD connected to one normal and two SC leads

We next consider the three terminal case, $\Gamma_N \neq 0$, with the additional normal lead. Specifically, we concentrate on the $\Delta \to \infty$ limit, where Δ is much larger than the other energy scales $\Delta \gg \max(\Gamma_S, \Gamma_N, U, |\epsilon_d|)$, and the quasi-particle excitations in the continuum energy region above the SC gap are projected out. Nevertheless, the essential physics of the low-energy transport is still preserved in this limit, and a static pair potential Δ_d is induced into the impurity site owing to the SC proximity effects [7, 10],

$$\Delta_d \equiv \Gamma_R e^{i\theta_R} + \Gamma_L e^{i\theta_L} = |\Delta_d| \, e^{i\theta_d} \,, \qquad |\Delta_d| = \Gamma_S \sqrt{1 - \mathcal{T}_0 \sin^2(\phi/2)} \,, \tag{3}$$

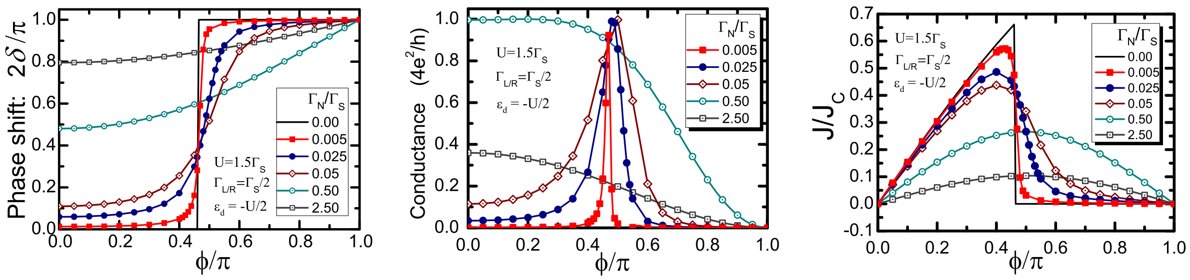

Figure 3. NRG results for the phase shift δ for the Bogoliubov particles (left), the conductance g_{NS} between the QD and normal lead (middle), and the Josephson current J (right), are plotted vs ϕ for several Γ_N in the large gap limit. The other parameters are chosen such that $U = 1.5\Gamma_S$, $\Gamma_L = \Gamma_R$ $(= \Gamma_S/2)$, and $\epsilon_d = -U/2$. Here, $J_C = e\Gamma_S/\hbar$ since $\Delta \to \infty$.

where $\mathcal{T}_0 = 4\Gamma_R\Gamma_L/(\Gamma_R+\Gamma_L)^2$. Carrying out the Bogoliubov transform with the angle Θ_B,

$$\cos\Theta_B = \frac{\xi_d}{E_A}, \qquad \sin\Theta_B = \frac{|\Delta_d|}{E_A}, \qquad E_A \equiv \sqrt{\xi_d^2 + |\Delta_d|^2}, \tag{4}$$

the Hamiltonian can be mapped onto the asymmetric Anderson model for the Bogoliubov particles, the Green's function for which takes the form $G(\omega) = [\omega - E_A + i\Gamma_N - \Sigma(\omega)]^{-1}$ [15]. The phase shift δ, defined by $G(0) = -|G(0)|e^{i\delta}$, determines the ground-state properties

$$\langle n_d\rangle - 1 = (2\delta/\pi - 1)\cos\Theta_B\,, \qquad \langle d_\downarrow d_\uparrow\rangle = \frac{1}{2}\,(2\delta/\pi - 1)\, e^{i\theta_d}\sin\Theta_B\,. \tag{5}$$

The conductance g_{NS} between the QD and normal lead, and the Josephson current J between the two SC leads can be determined by the three phase variables δ, Θ_B and ϕ,

$$g_{NS} = \frac{4e^2}{h}\sin^2\Theta_B\sin^2 2\delta, \qquad J = \frac{e\Gamma_S}{\hbar}\,\frac{\mathcal{T}_0\;|2\delta/\pi - 1|\,\sin\Theta_B\,\sin\phi}{2\sqrt{1-\mathcal{T}_0\sin^2(\phi/2)}}\,. \tag{6}$$

Furthermore, the renormalization factor $Z = [1 - \partial\Sigma/\partial\omega]^{-1}$, the position of the Andreev level $\widetilde{E}_A = \pm Z[E_A + \Sigma(0)]$, and the Wilson ratio R for the Bogoliubov particles can also be deduced from the local Fermi-liquid behavior, using the NRG.

The results are plotted *vs* ϕ in Figs. 3 and 4 for several values of Γ_N/Γ_S. The Coulomb interaction is chosen to be $U = 1.5\Gamma_S$, and for this value of U the QPT occurs in the $\Gamma_N \to 0$ limit at $\phi \simeq 0.46\pi$, where $E_A = U/2$. Note that $\mathcal{T}_0 = 1$ and $\xi_d = 0$ in the present case. The gapless excitations near the normal lead change the sharp QPT into a continuous crossover between the two different singlet states. We see in Fig. 3 that for $\phi \lesssim 0.46\pi$ the ground state is a local Cooper pairing consisting of a linear combination of the empty and doubly occupied impurity states with a small δ. On the other side, for $\phi \gtrsim 0.46\pi$, the ground state is a Kondo singlet with $\delta \simeq \pi/2$. At the transient region, the conductance g_{NS} has a peak, and the Josephson current J decreases rapidly to zero. Note that a weak current, which flows in the opposite direction as seen in Fig. 2 for the magnetic doublet state with finite Δ, is absent in the large gap limit. These features that are caused by the crossover are smeared as Γ_N increases.

Figure 4 shows the results for the renormalized parameters. We see for small Γ_N $(= 0.05\Gamma_S)$ that the parameters are strongly renormalized in the Kondo-singlet region for $\phi \gtrsim 0.46\pi$, where $Z \ll 1.0$, $R \simeq 2.0$, and the couple of the Andreev peaks $\widetilde{E}_A$ stay close to the Fermi level $\omega = 0$. In contrast, in the local Cooper-pairing region for $\phi \lesssim 0.46\pi$, the parameters are almost *not* renormalized $Z \simeq 1.0$, $R \simeq 1.1$, and the Andreev peaks situate away from the Fermi level. When the coupling Γ_N between the QD and normal lead is large, these two singlet states cannot be distinguished clearly.

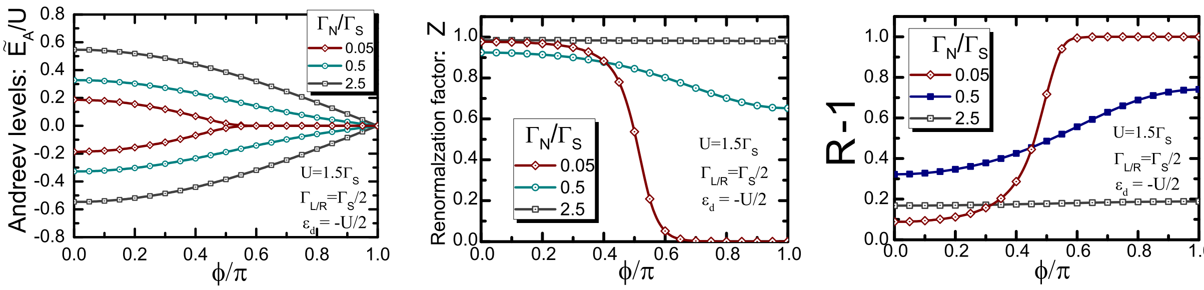

Figure 4. NRG results for the level position of renormalized Andreev resonance $\widetilde{E}_A$ (left), the renormalization factor Z (middle) and $R-1$ (right), where R is the Wilson ratio, are plotted *vs* ϕ in the large gap limit for $U = 1.5\Gamma_S$ and $\Gamma_L = \Gamma_R$ $(= \Gamma_S/2)$ for several Γ_N.

5. Summary

We have studied the crossover between the Kondo singlet and the local Copper pairing, which consists of the linear combination of the empty and doubly occupied impurity states, in the quantum dot coupled to the one normal and two SC leads. In this three terminal geometry the normalized parameters, which characterize the Fermi-liquid behavior of the Bogoliubov particles, vary depending on the Josephson phase ϕ. Therefore, the crossover can occur at finite Josephson phase ϕ_C. We calculated the phase shift δ and the renormalized parameters with the NRG approach, and observed numerically that the Andreev conductance between the dot and normal lead has a peak near the crossover region at $\phi \simeq \phi_C$. The Josephson current between the the two SC leads decreases rapidly in the strongly renormalized Kondo-singlet ground state for $\phi \gtrsim \phi_C$, where the Andreev states $\widetilde{E}_A$ stay close to the Fermi level. In the local Cooper-paring ground state at the opposite side $\phi \lesssim \phi_C$, the local Fermi-liquid parameters are not renormalized so much.

Acknowledgments

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References

[1] M. R. Gräber, T. Nussbaumer, W. Belzig, and C. Scönenberger, Nanotechnology **15**, S479 (2004).
[2] A. Eichler, M. Weiss, S. Oberholzer, C. Schönenberger, A. Levy Yeyati, J. C. Cuevas, and A. Martín-Rodero, Phys. Rev. Lett. **99**, 126602 (2007).
[3] T. Sand-Jespersen, J. Paaske, B. M. Andersen, K. Grove-Rasmussen, H. I. Jørgensen, M. Aagesen, C. B. Sørensen, P. E. Lindelof, K. Flensberg, and J. Nygård, Phys. Rev. Lett. **99**, 126603 (2007).
[4] R. S. Deacon, Yoichi Tanaka, A. Oiwa, R. Sakano, K. Yoshida, K. Shibata, K. Hirakawa, and S. Tarucha, Phys. Rev. B **81**, 121308 (2010).
[5] R. Fazio and R. Raimondi, Phys. Rev. Lett. **80**, 2913 (1998); **82**, 4950(E) (1999).
[6] A. A. Clerk, V. Ambegaokar, and S. Hershfield, Phys. Rev. B **61**, 3555 (2000).
[7] J. C. Cuevas, A. L. Yeyati, and A. Martín-Rodero, Phys. Rev. B **63**, 094515 (2001).
[8] Y. Avishai, A. Golub, and A. D. Zaikin, Phys. Rev. B **63**, 134515 (2001).
[9] M. Governale, M. G. Pala and J. König, Phys. Rev. B **77**, 134513 (2008).
[10] Yoshihide Tanaka, A. Oguri, and A. C. Hewson, New J. Phys. **9**, 115 (2007); **10**, 029801(E) (2008).
[11] J. Bauer, A. Oguri, and A. C. Hewson, J. Phys.: Condes. Mat. **19**, 486211 (2007).
[12] C. Karrasch, A. Oguri, and V. Meden, Phys. Rev. B **77**, 024517 (2008).
[13] T. Hecht, A. Weichselbaum, J. von Delft, R. Bulla, J. Phys.: Condes. **20**, 275213 (2008).
[14] T. Meng, S. Florens, and P. Simon, Phys. Rev. B **79** 224521 (2009).
[15] Yoichi Tanaka, N. Kawakami, and A. Oguri, J. Phys. Soc. Jpn. **76**, 074701 (2007); **77**, 098001(E) (2008).